ECONOMIC AND REGULATORY CAPITAL
WHAT IS THE DIFFERENCE?

Abel Elizalde y Rafael Repullo

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Abstract

This paper analyzes the determinants of regulatory capital (the minimum required by regulation) and economic capital (the capital that shareholders would choose in absence of regulation) in the context of the single risk factor model that underlies the New Basel Capital Accord (Basel II). The results show that economic and regulatory capital do not depend on the same set of variables and do not react in the same way to changes in their common determinants. For plausible parameter values, they are both increasing in the loans' probability of default and loss given default, but variables that affect economic but not regulatory capital, such as the intermediation margin and the cost of capital, can move them significantly apart. The results also show that market discipline, proxied by the coverage of deposit insurance, increases economic capital, although the effect is generally small.

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1   Introduction

Economic and regulatory capital are two terms frequently used in the analysis of the new framework for bank capital regulation recently finalized by the Basel Committee on Banking Supervision (2004), known as Basel II. In particular, many discussions preceding the publication of the new regulation have highlighted the objective of bringing regulatory capital closer to economic capital. For example, Gordy and Howells (2004, p.1) state that "the primary objective under Pillar 1 (of Basel II) is better alignment of regulatory capital requirements with 'economic capital' demanded by investors and counterparties."

To compare economic and regulatory capital we first must clarify the meaning of each term. Although not always precisely defined, *economic capital* may be understood, and this is the definition that we will use hereafter, as the capital level that bank shareholders would choose in absence of capital regulation. In contrast, the definition of *regulatory capital* is clear: it is the minimum capital required by the regulator, which in this paper we identify with the capital charges in the internal ratings based (IRB) approach of Basel II.[1]

The purpose of this paper is to analyze the differences between economic and regulatory capital in the context of the asymptotic single risk factor model that underlies the IRB capital requirements of Basel II. To compute economic capital we use a dynamic model in which the bank shareholders choose, at the beginning of each period, the level of capital in order to maximize the market value of the bank, taking into account the possibility that the bank be closed if the losses during the period exceed the initial level of capital.[2] Thus economic capital trades-off the costs of funding the bank with costly equity against the benefits of reducing the probability of losing its franchise value, which appears as a key endogenous value in the bank's maximization problem. It is important to stress that in our model bank shareholders only choose the level of capital, not the risk characteristics of its loan portfolio which are taken as given.

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[1]In principle, regulatory capital should be derived from the maximization of a social welfare function that takes into account both the costs (e.g. the increase in the cost of credit) and the benefits (e.g. the reduction in the probability of bank failure) of capital regulation; see Repullo and Suarez (2004) for a discussion of this issue.

[2]This could be justified by assuming that either the bank supervisor withdraws its license or that a bank run takes place before the shareholders can raise new equity to cover the losses.

We show that economic and regulatory capital do not depend on the same variables: the former (but not the latter) depends on the intermediation margin and the cost of bank capital, while the latter (but not the former) depends on the confidence level set by the regulator. Moreover, economic and regulatory capital do not respond in the same manner to changes in the common variables that affect them, such as the loans' probability of default, loss given default, and exposure to the systematic risk factor.

Due to the difficulty of obtaining analytical results for economic capital, we use a numerical procedure to compute it. The results show that Basel II regulatory capital only approaches economic capital for a limited, although reasonable, range of parameter values. The results also show that the relative position of economic and regulatory capital is mainly determined by the cost of bank capital: economic capital is higher (lower) than regulatory capital for values of the cost of bank capital lower (higher) than a given critical value. Another key variable in the shareholders' economic capital decision is the intermediation margin, which has two opposite effects. On the one hand, a higher margin increases the bank's franchise value and consequently shareholders' incentives to contribute capital. On the other hand, a higher margin increases bank revenues and therefore reduces the role of capital as a buffer to absorb future losses, acting as a substitute of economic capital. We show that the net effect of the intermediation margin on economic capital is positive in very competitive banking markets and negative otherwise.[3]

The numerical results also show that increases in the loans' probability of default, loss given default, and exposure to the systematic risk factor increase regulatory capital, while they only increase economic capital for a range of (reasonable) values of these variables. At any rate, the correlation between economic and regulatory capital in Basel II is clearly higher than in the 1988 Basel Accord (Basel I), where regulatory capital was practically independent of risk.[4]

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[3]It should be noted that the aim of this paper is to analyze the determinants of economic capital, not of the probability of bank failure. The fact that a given variable increases economic capital does not mean that it necessarily reduces the probability of failure. For example, a higher intermediation margin might reduce both economic capital and the probability of failure.

[4]Basel I required a minimum capital equal to 8% of the bank's risk weighted assets. Two basic criteria were used to compute these weights: the institutional nature of the borrower and the collateral provided. In particular, the weights were 0% for sovereign risks with OECD countries, 20% for interbank risks, 50% for mortgages, and 100% for all other risks.

The model proposed in the paper allows us to analyze the effect of market discipline, proxied by the coverage of deposit insurance, on economic capital. We consider two alternative scenarios: one in which depositors are fully insured and where the deposit interest rate is equal to the risk free rate, and another one in which depositors are uninsured. In both cases, depositors require an interest rate such that the expected return of their investment is equal to the risk free rate. The results suggest that measures aimed at increasing market discipline, such as those contemplated in Pillar 3 of Basel II, have a positive effect on economic capital, though its magnitude is generally small, except in very competitive markets for high risk loans.

It is important to emphasize some of the limitations of our analysis, such as the assumption that bank risk level is exogenous or the use of the asymptotic single risk factor model. The inclusion of the bank's level of risk as an endogenous variable, together with economic capital, in the shareholders' maximization problem, as well as the analysis of more complex models of bank risk are left for future research.

The academic literature on this topic is very small. From a theoretical perspective, the most interesting paper (which is the basis of our analysis of economic capital) is Suarez (1994), who constructs a dynamic model of bank behavior in which shareholders choose the capital structure as well as the asset risk. From an empirical perspective, Flannery and Rangan (2002) analyze the relationship between regulatory and actual bank capital between 1986 and 2000 for a sample of US banks. The authors conclude that the increase in regulatory capital during the first part of the 1990s could explain the increase in the capital levels of the banking industry during those years, but that the additional increase in capital in second part of the 1990s is mainly driven by market discipline.

This paper is organized as follows. Section 2 presents the model and characterizes the determinants of regulatory and economic capital. Section 3 derives the numerical results, and Section 4 concludes. Appendix A discusses the comparative statics of economic capital, and Appendix B contains a proof of the negative relationship between bank capital and the interest rate on uninsured deposits.

2 The Model

Consider a bank that at the beginning of each period $t = 0, 1, 2, ...$ in which it is open has an asset size that is normalized to 1. The bank is funded with *deposits*, $1 - k_t$, that have an interest rate $c \geq 0$, and *capital*, k_t, that requires an expected return $\delta > c$. The bank is owned by risk-neutral shareholders who enjoy limited liability and, in the absence of minimum capital regulation, choose the capital level $k_t \in [0, 1]$. To simplify the presentation, we assume that there are no intermediation costs.

In each period t in which the bank is open, its assets are invested in a portfolio of loans paying an exogenously fixed interest rate r. The return of this investment is stochastic: a random fraction $p_t \in [0, 1]$ of these loans default, in which case the bank loses the interest r as well as a fraction $\lambda \in [0, 1]$ of the principal. Therefore, the bank gets $1 + r$ from the fraction $1 - p_t$ of the loans that do not default, and it recovers $1 - \lambda$ from the fraction of defaulted loans, so its asset value at the end of period t is given by

$$a_t = (1 - p_t)(1 + r) + p_t(1 - \lambda). \tag{1}$$

Since the bank has to pay depositors an amount $(1 - k_t)(1 + c)$, its capital level at the end of period t is

$$k'_t = a_t - (1 - k_t)(1 + c) = k_t + (1 - p_t)r - p_t\lambda - (1 - k_t)c. \tag{2}$$

There exists a supervisor that, at the end of each period t, verifies the bank's capital k'_t and withdraws its license whenever $k'_t < 0$, that is when the losses $p_t\lambda + (1 - k_t)c - (1 - p_t)r$ exceed the initial capital level k_t. In that case, the assumption of limited liability implies that bank shareholders do not make any payment to the depositors. From the definition (2) of k'_t is immediate to verify that the bank will fail if

$$p_t > p(k_t) = \min\left\{\frac{k_t + r - (1 - k_t)c}{r + \lambda}, 1\right\}, \tag{3}$$

that is, if the *default rate* p_t exceeds the critical level $p(k_t)$. Notice that $p(k_t)$ is increasing in the initial capital level k_t, with $p(k_t) = 1$ when k_t is greater than or equal to the sum of the loss given default λ and the cost of deposits $(1 - k_t)c$, in which case the probability of bank failure is zero.

Let $I_{t+1} \in \{0,1\}$ be a random variable that indicates whether the bank is closed ($I_{t+1} = 0$) or open ($I_{t+1} = 1$) at the beginning of period $t+1$. The closure rule can be formalized as

$$I_{t+1} = I_t h(k_t'), \tag{4}$$

where

$$h(k_t') = \begin{cases} 0, & \text{if} \quad k_t' < 0, \\ 1, & \text{if} \quad k_t' \geq 0. \end{cases} \tag{5}$$

Thus when $k_t' < 0$ the bank's license is permanently withdrawn by the supervisor.[5]

We assume that the probability distribution of the default rate p_t is the one derived from the *single risk factor* model of Vasicek (2002), which underlies the IRB capital requirements of Basel II; see also Gordy (2003). Its distribution function is given by

$$F(p_t) = N \left(\frac{\sqrt{1-\rho}\, N^{-1}(p_t) - N^{-1}(\overline{p})}{\sqrt{\rho}} \right), \tag{6}$$

where $N(\cdot)$ is the distribution function of a standard normal random variable, $\overline{p} \in [0,1]$ is the loans' (unconditional) probability of default, and $\rho \in [0,1]$ is their exposure to the systematic risk factor: when $\rho = 0$ defaults are statistically independent, so $p_t = \overline{p}$ with probability 1, while when $\rho = 1$ defaults are perfectly correlated, so $p_t = 0$ with probability $1 - \overline{p}$, and $p_t = 1$ with probability $\overline{p}$. We also assume that p_t is independent over time.

The distribution function $F(p_t)$ is increasing, with $F(0) = N(-\infty) = 0$ and $F(1) = N(\infty) = 1$. Moreover, it can be shown that

$$E(p_t) = \int_0^1 p_t \, dF(p_t) = \overline{p}$$

and

$$Var(p_t) = \int_0^1 (p_t - \overline{p})^2 \, dF(p_t) = N_2(N^{-1}(\overline{p}), N^{-1}(\overline{p}); \rho) - \overline{p}^2,$$

where $N_2(\cdot, \cdot; \rho)$ is the distribution function of a zero mean bivariate normal random variable with standard deviation equal to one and correlation coefficient ρ; see Vasicek (2002, p.161). Therefore, the expected value of the default rate is precisely the probability of default $\overline{p}$, while its variance is increasing with the correlation parameter ρ, with $Var(p_t) = 0$ for $\rho = 0$ and $Var(p_t) = \overline{p}(1 - \overline{p})$ for $\rho = 1$.

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[5]Suarez (1994) analyzes an alternative closure rule in which the supervisor allows the bank shareholders to recapitalize the bank, and hence prevent its closure, when $k_t' < 0$.

2.1 Regulatory capital

According to the internal ratings based (IRB) approach of Basel II bank capital must cover losses due to loan defaults with a given probability (or confidence level) $\alpha = 99,9\%$.

In particular, given the distribution function $F(p_t)$ of the default rate p_t, let $\widehat{p}$ be the critical value such that

$$\Pr(p_t \leq \widehat{p}) = F(\widehat{p}) = \alpha,$$

which implies $\widehat{p} = F^{-1}(\alpha)$. Making use of (6), we then obtain the capital requirement

$$\widehat{k} = \lambda \widehat{p} = \lambda N \left(\frac{N^{-1}(\overline{p}) + \sqrt{\rho} \, N^{-1}(\alpha)}{\sqrt{1 - \rho}} \right). \tag{7}$$

This is the formula that appears in BCBS (2004, paragraph 272), except for the fact that we are assuming a one year maturity (which implies a maturity adjustment factor equal to 1) and that the correlation parameter ρ is in Basel II a decreasing increasing function of the default probability $\overline{p}$.[6] Also, Basel II establishes that expected losses, $\lambda \overline{p}$, should be covered with general loan loss provisions, while the remaining charge, $\lambda(\widehat{p} - \overline{p})$, should be covered with capital, but from the perspective of our analysis provisions are just another form of equity capital.

From this expression we can immediately identify the *determinants of regulatory capital* $\widehat{k}$, which are the loans' probability of default $\overline{p}$, loss given default λ, and exposure to systematic risk ρ, and the confidence level α set by the regulator.

To analyze the effects on the level of regulatory capital $\widehat{k}$ of changes in its determinants, we differentiate the function (7) which gives

$$\frac{\partial \widehat{k}}{\partial \overline{p}} > 0, \quad \frac{\partial \widehat{k}}{\partial \lambda} > 0 \quad \text{and} \quad \frac{\partial \widehat{k}}{\partial \alpha} > 0.$$

Moreover, we also get

$$\frac{\partial \widehat{k}}{\partial \rho} > 0 \quad \text{if and only if} \quad \alpha > 1 - N(\sqrt{\rho} \, N^{-1}(\overline{p})),$$

which holds for a sufficiently high confidence level α.[7] Therefore, we conclude that regulatory capital $\widehat{k}$ is an increasing function of its four determinants.

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[6]In particular, for corporate, sovereign, and bank exposures Basel II establishes that $\rho = 0.24 - 0.12(1 - e^{-50\overline{p}})/(1 - e^{-50})$.

[7]For example, for $\overline{p} = 0.02$ and $\rho = 0.2$ it follows that $1 - N(\sqrt{\rho} \, N^{-1}(\overline{p})) = 0.8208$.

2.2 Economic capital

To derive the level of capital chosen by the bank shareholders in the absence of minimum capital regulation we solve a dynamic programming problem in which the state variable $I_t \in \{0, 1\}$ indicates whether the bank is closed ($I_t = 0$) or open ($I_t = 1$) at the beginning of period t.

The value function of this problem, $V(I_t)$, is implicitly defined by the Bellman equation

$$V(I_t) = \max_{k_t \in [0,1]} I_t \left[-k_t + \frac{1}{1+\delta} E\left[\max\{k'_t, 0\} + V(I_{t+1}) \right] \right]. \tag{8}$$

According to this expression, $V(0) = 0$, that is the value of a bank that is closed is zero. On the other hand, $V(1)$ is the *franchise value* of a bank that is open. This value, which henceforth will simply be denoted by V, results from maximizing, with respect to capital k_t, an objective function that has three terms: the first one, with negative sign, is the capital contribution of shareholders at the beginning of period t; the second one is the discounted expected payoff at the end of period t, which under limited liability is equal to $\max\{k'_t, 0\}$; and the third one is the discounted expected value of remaining open in period $t + 1$, and therefore of having the possibility of receiving a stream of future dividends. Notice that the discount rate used in the last two terms is the return required by bank shareholders or cost of capital δ.

Therefore, assuming that $I_t = 1$, there are two possible scenarios at the end of period t: if $k'_t < 0$ the bank fails and the shareholders get a final payoff of zero; and if $k'_t \geq 0$ the bank remains open in period $t + 1$ and the shareholders receive a dividend payment (or make a capital contribution, depending on the sign) of $k'_t - k_{t+1}$, that is the difference between the capital at the end of period t and the capital that they would like to keep in the bank for period $t + 1$.

From the definition (2) of the capital level at the end of the period k'_t, when the initial capital verifies

$$k_t \geq k_{\max} = \frac{\lambda + c}{1 + c},$$

the probability of bank failure is zero, so

$$E\left[\max\{k'_t, 0\} + V(I_{t+1}) \right] = k_t + (1 - \overline{p})r - \overline{p}\lambda - (1 - k_t)c + V.$$

In this case the derivative with respect to k_t of the expression in the right hand side of the Bellman equation (8) is negative, which implies that bank shareholders will

never choose a capital level k_t higher than the critical level $k_{\max}$. This result is easy to explain. Bank shareholders might be willing to contribute capital, instead of funding the bank with cheaper deposits, as long as capital provides a buffer that reduces the probability of failure, and consequently increases the probability of receiving a stream of future dividends. However, if $k_t \geq k_{\max}$ capital covers the bank loses at the end of period t, even if 100% of the loans in its portfolio default, which means that any additional capital will only increase the bank's funding costs without reducing its probability of failure (which is zero). It follows then that bank shareholders will never want to provide $k_t > k_{\max}$, so we can limit the range of values for k_t in the Bellman equation (8) to the interval $[0, k_{\max}]$.

Substituting the definition (2) of k'_t in $E\left[\max\{k'_t, 0\}\right]$, and integrating by parts, taking into account the restriction $k_t \leq k_{\max}$, yields

$$E\left[\max\{k'_t, 0\}\right] = (\lambda + r) \int_0^{p(k_t)} F(p_t)\ dp_t.$$

Moreover, (4) and (5) imply

$$E\left[V(I_{t+1})\right] = \Pr(k'_t \geq 0)V = F(p(k_t))V.$$

Therefore, since the shareholders' maximization problem is identical in all periods, we can leave out the temporal subindex t and rewrite the Bellman equation (8) as

$$V = \max_{k \in [0, k_{\max}]} G(k, V), \tag{9}$$

where

$$G(k, V) = -k + \frac{1}{1+\delta}\left[(\lambda + r)\int_0^{p(k)} F(p)\ dp + F(p(k))V\right]. \tag{10}$$

The solution of this equation gives the level of economic capital k^* that bank shareholders would like to hold in the absence of minimum capital requirements, as well as the bank's franchise value V. In addition, this equation allows us to identify the *determinants of economic capital* k^*, which are the loans' probability of default $\bar{p}$, loss given default λ, and exposure to systematic risk ρ, the loan rate r, the deposit rate c, and the cost of bank capital δ. Notice that these last three variables do not affect regulatory capital $\widehat{k}$, while the confidence level α set by the regulator does not affect economic capital k^*.

Appendix A shows that economic capital can be at the corner $k^* = 0$, and that if there is an interior solution comparative static results cannot be derived analytically, except for the deposit rate c and the cost of capital δ, for which we obtain

$$\frac{\partial k^*}{\partial c} < 0 \quad \text{and} \quad \frac{\partial k^*}{\partial \delta} < 0.$$

In other words, the higher the bank's deposit and equity funding costs the lower the capital provided by its shareholders. Due to the difficulty of obtaining analytical results, the discussion of the effects of the determinants of economic capital will be based on numerical solutions.

To conclude, it is important to highlight the different determinants of economic and regulatory capital. Both of them depend on the loans' probability of default $\overline{p}$, loss given default λ, and exposure to systematic risk ρ. However, while an increase in any of these variables increases regulatory capital, its effect on economic capital is, in general, ambiguous. Moreover, economic capital depends on the loan rate r, the deposit rate c, and the cost of bank capital δ, whereas regulatory capital depends on the confidence level α set by the regulator.

3 Numerical Results

This section compares the values of economic capital k^* and regulatory capital $\widehat{k}$ obtained by solving the Bellman equation (9) and computing the IRB formula (7), respectively, for different values of the parameters of the model.[8]

For the benchmark case, we assume a probability of default $\overline{p}$ of 2%, a loss given default λ of 45%, and an exposure to systematic risk ρ of 20%. For computing regulatory capital, we use the confidence level $\alpha = 99,9\%$ set in Basel II.

With regard to the loan rate r, instead of taking it as exogenous, we assume that it is determined according to the equation

$$(1 - \overline{p})r - \overline{p}\lambda = \mu, \tag{11}$$

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[8]The Bellman equation is solved by an iterative procedure: given an initial franchise value V_0 we compute $V_1 = \max_k G(k, V_0)$, and iterate the process until convergence to a value V. Economic capital is then given by $k^* = \arg\max_k G(k, V)$.

that equals the expected return of a loan, $(1 - \overline{p})r - \overline{p}\lambda$, to a margin μ over the risk free rate, which is normalized to zero.[9] Rearranging (11) we obtain

$$r = \frac{\mu + \overline{p}\lambda}{(1 - \overline{p})},$$

so the loan rate r is an increasing function of the probability of default $\overline{p}$, the loss given default λ, and the intemediation margin μ. In the benchmark case we take a value for μ of 0.5%.

For the deposit rate c, we assume that the return required by depositors is equal to the risk free rate, which has been normalized to zero, and we consider two alternative scenarios. In the first one depositors are fully insured by a deposit insurance agency, and therefore (ignoring the deposit insurance premium) the deposit rate c is equal to the risk free rate, i.e. $c = 0$. In the second one depositors are uninsured, so under the assumption of risk neutrality the deposit rate c has to verify the participation constraint

$$E\left[\min\left\{a, (1 - k)(1 + c)\right\}\right] = 1 - k. \tag{12}$$

To understand this equation notice that when the value of the bank's assets a is greater than or equal to the deposits' principal and interest, that is when $k' = a - (1 - k)(1 + c) \geq 0$, depositors receive $(1 - k)(1 + c)$, whereas when $k' < 0$ the bank is closed by the supervisor and depositors receive the liquidation value of the bank, which ignoring bankruptcy costs is equal to a. Thus the left hand side of (12) is the expected value of the depositors' claim at the end of each period, while the right hand side is the gross return that they require on their investment.

The last parameter that has to be specified is the return δ required by bank shareholders, which in the benchmark case will be set equal to 2%. Since we have normalized the risk free rate to zero, this value should be interpreted as a spread over the risk free rate.

Table 1 summarizes the parameter values in the benchmark case, as well as the range of values for which economic and regulatory capital will be computed (keeping the rest of the parameters at their benchmark levels).

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[9]Equation (11) is an approximation to the loan rate equation derived by Repullo and Suarez (2004) for a perfect competition model.

Parameter	Benchmark case	Range of values
Probability of default $\bar{p}$	2%	$0 - 20\%$
Intermediation margin μ	0.5%	$0 - 5\%$
Cost of bank capital δ	2%	$0 - 10\%$
Exposure to systematic risk ρ	20%	$0 - 50\%$
Loss given default λ	45%	$0 - 100\%$

Table 1. Parameter values used in the numerical exercise.

When discussing the effects of the different parameters on economic capital k^* we will distinguish between the cases of insured and uninsured deposits. To understand the differences between these two cases it is important to analyze the effect of bank capital on the interest rate required by uninsured depositors derived from the participation constraint (12). Appendix B shows that this equation has a unique solution $c(k) \geq 0$ for all k and that $c'(k) < 0$, except for $k \geq \lambda$ in which case $c'(k) = c(k) = 0$.

Figure 1 represents the cost of uninsured deposits c as a function of the capital level k, that is the function $c(k)$, for the benchmark case parameters, $\bar{p} = 2\%$ and $\mu = 0,5\%$, as well as the effects of an increase in $\bar{p}$ and in μ. The negative effect of k on the uninsured deposit rate c is significant for small values of k, for which the probability of bank failure is relatively high. An increase in the intermediation margin μ from 0.5% to 1% reduces this probability and consequently the deposit rate c, whereas an increase in the probability of default $\bar{p}$ from 2% to 5% has the opposite effect.

Figure 2 plots, for two different values of the intermediation margin $\mu = 0.5\%$ and 1%, regulatory capital $\widehat{k}$, as well as economic capital with insured and uninsured deposits, k_i^* and k_u^*, as a function of the loans' probability of default $\bar{p}$. As discussed in Section 2, an increase in $\bar{p}$ increases regulatory capital, but has an ambiguous effect on economic capital. In particular, the left panel of Figure 2 shows that for an intermediation margin $\mu = 0.5\%$ economic capital with insured deposits k_i^* is increasing in the probability of default for values of $\bar{p}$ smaller than 10%, it is decreasing for values of $\bar{p}$ between 10% and 17%, and it jumps to the corner solution $k_i^* = 0$ for higher values of $\bar{p}$. Economic capital with uninsured deposits k_u^* is also first increasing and then decreasing in the probability of default $\bar{p}$, although for much higher levels of $\bar{p}$.
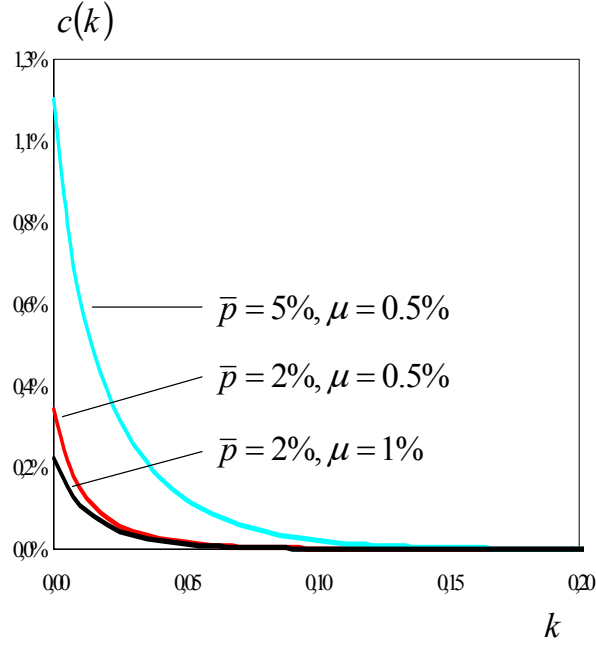
Figure 1: Effect of bank capital on the uninsured deposits' interest rate

The reason why the relationship between the probability of default $\overline{p}$ and economic capital is nonmonotonic is that, for high values of $\overline{p}$, the bank probability of failure is so high that bank shareholders prefer to reduce (even to zero) their capital contribution. This result holds for relative higher levels of the probability of default $\overline{p}$ when deposits are uninsured because, as illustrated in Figure 1, the uninsured deposits' interest rate is a decreasing function of the level of capital, which implies that shareholders have an additional incentive to provide capital. It should be noted, however, that these somewhat surprising results only hold for implausibly high values of the probability of default $\overline{p}$, so their practical relevance may be limited.

Economic capital with insured deposits k_i^* is always below economic capital with uninsured deposits k_u^*, so we conclude that the market discipline introduced by the need to provide the required expected return to the uninsured depositors implies higher bank capital. Figure 2 shows that this effect is more important when the loans' probability of default $\overline{p}$ is higher and the intermediation margin μ is lower, that is in very competitive markets for high risk loans, which is explained by the higher impact in these markets of the capital level k on the uninsured deposits' interest rate c noted

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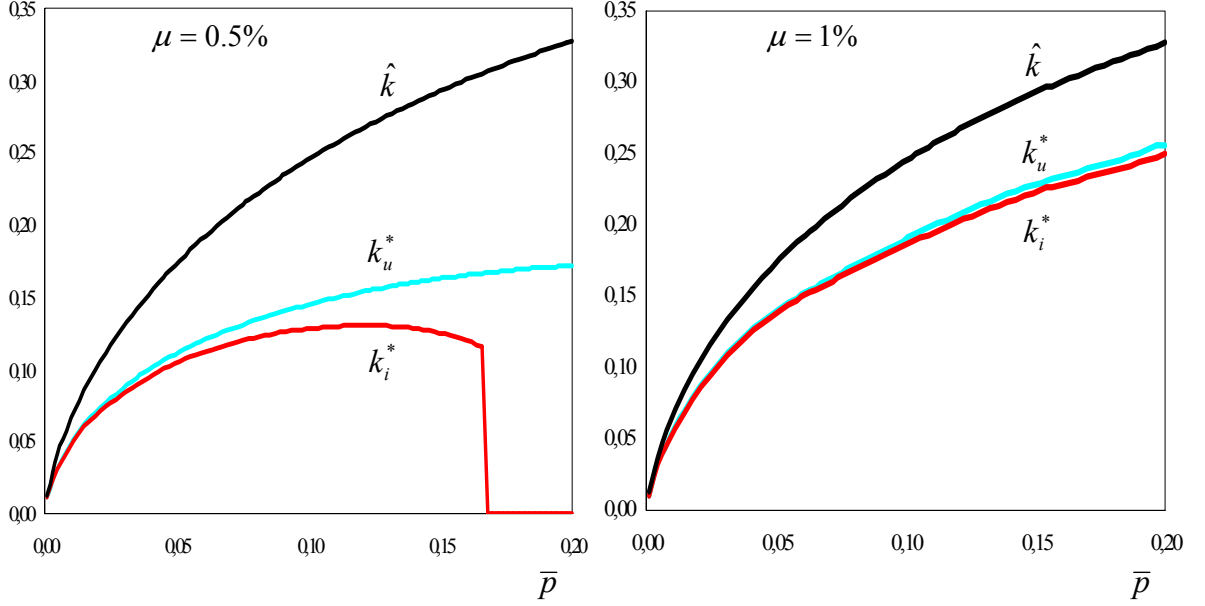
Figure 2: Effect of the probability of default on economic and regulatory capital

above. As we shall see later, the difference between economic capital with insured and uninsured deposits, k_i^* and k_u^*, is also increasing in the loans' loss given default λ and exposure to systematic risk ρ, and in the cost of bank capital δ.

The fact that $k_i^* \leq k_u^*$ might seem surprising in the light of the result in Appendix A showing that economic capital is decreasing in the deposit rate ($\partial k^*/\partial c < 0$). However the explanation is quite simple: in the model with uninsured deposits the deposit rate c is not only higher than in the model with insured deposits, but it is also decreasing in the capital level k, which increases shareholders' incentives to provide capital in order to reduce the cost of uninsured deposits.

Figure 2 also shows that the differences between regulatory and economic capital (with and without insured deposits) is mainly determined by the intermediation margin μ. In particular, comparing the left and right panels of Figure 2 we conclude that an increase in the intermediation margin μ increases economic capital both with insured and uninsured deposits, k_i^* and k_u^*, and reduces the distance between them. The first result stems from the fact that a higher intermediation margin μ implies a higher franchise value V, so shareholders have greater incentives to provide capital in
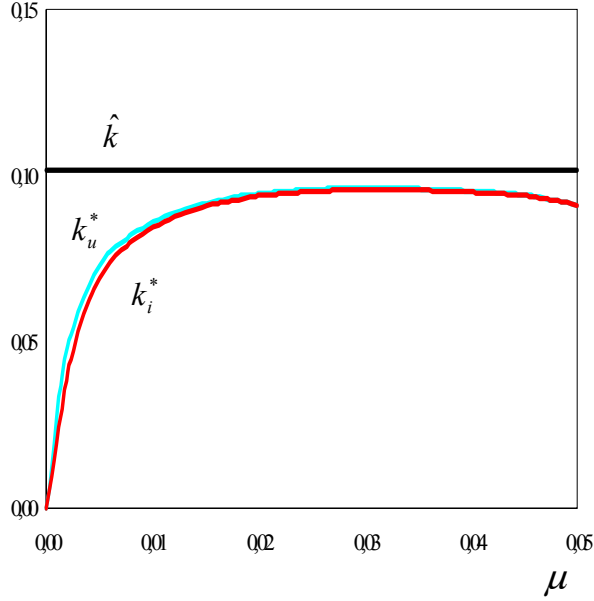
Figure 3: Effect of the intermediation margin on economic and regulatory capital

order to preserve it. The second result holds because a higher intermediation margin μ increases bank solvency and therefore reduces the interest rate on uninsured deposits, bringing it closer to the interest rate on insured deposits.

It is important to note that the intermediation margin μ has two opposite effects on economic capital. On the one hand, as we have already mentioned, a higher intermediation margin μ increases the bank's franchise value V, and therefore shareholders' incentives to provide capital. On the other hand, by assumption (11), a higher intermediation margin μ increases the loan rate r, which increases the asset value a at the end of the period and reduces the need to hold capital in order to protect the bank's franchise value V. From this perspective, economic capital k^* and the intermediation margin μ are substitutes, which may account for a possible negative relationship between them.

Figure 3 represents regulatory capital $\widehat{k}$, as well as economic capital with insured and uninsured deposits, k_i^* and k_u^*, as functions of the intermediation margin μ. For values of the intermediation margin μ below 3%, increases in the margin increase both levels of economic capital, bringing them closer to regulatory capital (which
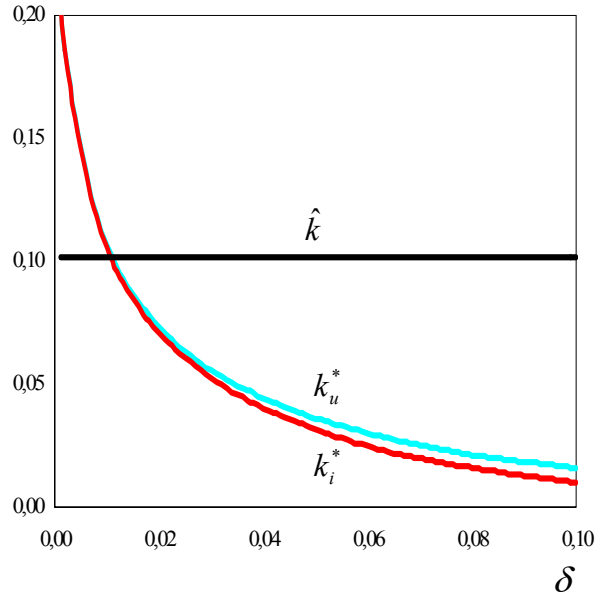
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Figure 4: Effect of the cost of bank capital on economic and regulatory capital

does not vary with μ), but the relationship becomes negative for higher values of the intermediation margin μ. Thus, for relatively competitive banking markets, the positive effect of the intermediation margin μ on economic capital k^*, via an increase in the bank's franchise value V, outweights its negative effect, via the substitution between economic capital k^* and the margin μ, while for oligopolistic markets the negative effect dominates.

In all cases analyzed so far, we have found economic capital below regulatory capital. This is mainly due to our benchmark parameter value for the cost of bank capital δ. Figure 4 represents regulatory capital $\widehat{k}$, as well as economic capital with insured and uninsured deposits, k_i^* and k_u^*, as functions of the cost of capital δ. As shown in Appendix A, economic capital is a decreasing function of the cost of capital $(\partial k^*/\partial \delta < 0)$. Moreover, for values of the cost of capital δ below 1%, both levels of economic capital, with and without insured deposits, are above regulatory capital. The reason for this is obvious: the lower the cost of capital δ, the higher the incentives of bank shareholders to contribute capital. In fact, for values of δ sufficiently close to zero, shareholders choose capital levels that effectively guarantee the bank's survival
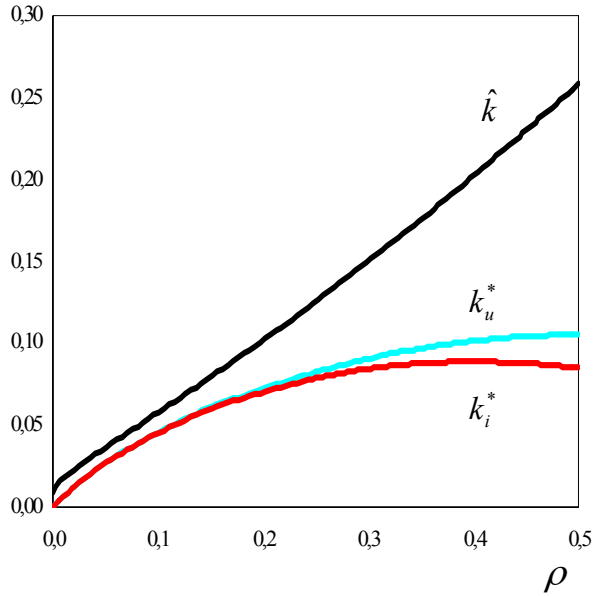
Figure 5: Effect of the exposure to systematic risk on economic and regulatory capital

regardless of the fraction of the loans in its portfolio that default.

Figure 5 plots regulatory capital $\widehat{k}$, as well as economic capital with insured and uninsured deposits, k_i^* and k_u^*, as functions of the loans' exposure to systematic risk ρ. As discussed in Section 2, the loans' exposure to systematic risk ρ is directly related with the variance of the default rate, so for a given probability of default $\overline{p}$, it is a measure of the risk of the loan portfolio. This explains why regulatory capital $\widehat{k}$ is an increasing function of ρ. Also, with respect to economic capital, its relationship with the exposure to systematic risk ρ resembles that of the probability of default $\overline{p}$ in Figure 2, first increasing and then decreasing.

Finally, Figure 6 plots regulatory capital $\widehat{k}$, as well as economic capital with insured and uninsured deposits, k_i^* and k_u^*, as functions of the bank loans' loss given default λ. According to the IRB formula (7), regulatory capital $\widehat{k}$ is a linear function of the loss given default λ. On the other hand, while the effect of λ on economic capital is positive in Figure 6, as discussed in Section 2, this is not true in general. For example, if the probability of default $\overline{p}$ and the cost of capital δ equal 5%, k_i^* and
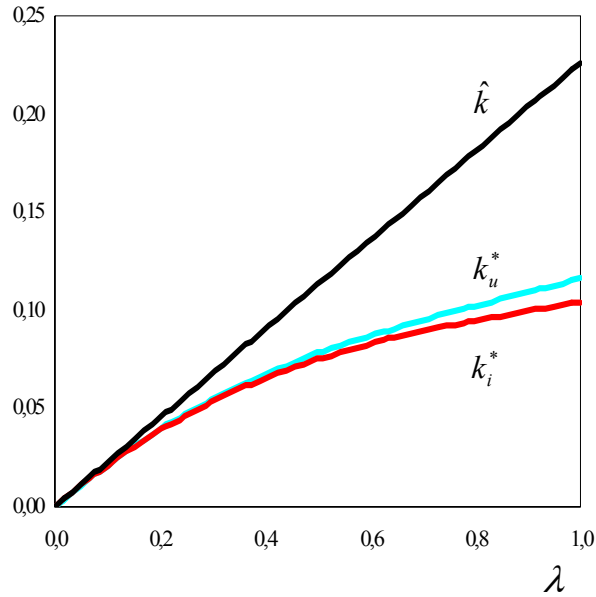
Figure 6: Effect of the loss given default on economic and regulatory capital

k_u^* start decreasing for values of λ of 30% and 52%, respectively.[10]

To sum up, we have found that both regulatory and economic capital depend positively on the loans' probability of default, loss given default, and exposure to systematic risk for reasonable values of these variables. However, variables that only affect economic capital, such as the intermediation margin and the cost of capital, may significantly move it away from regulatory capital. We have also found that market discipline, proxied by the coverage of deposit insurance, has a positive impact on economic capital, but the effect is in general small and very sensitive to the values of the rest of the determinants of economic capital.

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[10] Altman, Brady, Resti and Sironi (2005) find a strong positive correlation between the probability of default $\bar{p}$ and the loss given default λ. If we incorporated this relationship, our results regarding the difference between economic and regulatory capital as well as the difference between economic capital with and without insured deposits would be clearer, because (as illustrated in Figures 2 and 6) the effects of both variables are very similar.

4   Conclusions

Our analysis of the determinants of economic and regulatory capital for a bank whose loan default rates are described by the asymptotic single risk factor model that underlies the capital charges in the IRB approach of Basel II shows that there does not exist a direct relationship between both capital levels.

First, economic and regulatory capital do not depend on the same variables: regulatory capital (but not economic capital) depends on the confidence level set by the regulator, while economic (but not regulatory capital) depend on the intermediation margin and the cost of bank capital. These last two variables play a key role in determining the differences between them. Economic capital is only above regulatory capital for low values of the cost of capital, and when this cost increases, the former quickly falls below the latter. The effect of the intermediation margin on economic capital is only positive in fairly competitive credit markets, which is explained by the existence of two opposite effects: on the one hand, a higher margin increases the bank's franchise value, and hence shareholders' incentives to contribute capital in order to preserve it, but on the other hand a higher margin provides a source of income that reduces the need to hold capital as a buffer against losses. The first (positive) effect outweights the second (negative) in sufficiently competitive credit markets. Therefore, changes in the market power of banks, due for example to public policies, may have very different effects on economic capital depending on the initial level of competition.

Second, variables that affect both economic and regulatory capital, such as the loans' probability of default, loss given default, and exposure to systematic risk, have a positive impact on both capital levels for reasonable values of these variables. However, when they reach certain critical values their effect on economic capital becomes negative, increasing the gap with regulatory capital.

Finally, the comparison of economic capital with insured and uninsured deposits reveals that, even though the latter is never below the former, their differences are, in general, small and very sensitive to the values of the rest of the determinants of economic capital. Therefore, the effects of policies aimed at increasing market discipline, such as those contemplated in Pillar 3 of Basel II, may be limited, except in very competitive markets for high risk loans.

Appendix

A   Comparative statics of economic capital

This Appendix discusses the effects on economic capital k^* of changes in its determinants, namely the loans' probability of default $\bar{p}$, loss given default λ, and exposure to systematic risk ρ, the loan rate r, the deposit rate c, and the cost of bank capital δ. It is shown that only for the two last variables one can analytically derive the sign of their effect on economic capital.

To this end we first analyze the properties of the function $G(k, V)$ in the Bellman equation (10). Its derivatives with respect to k are given by

$$\frac{\partial G}{\partial k} = -1 + \frac{1+c}{1+\delta} \left[F(p(k)) + \frac{f(p(k))V}{\lambda + r} \right], \tag{13}$$

$$\frac{\partial^2 G}{\partial k^2} = \frac{(1+c)^2}{(1+\delta)(\lambda + r)} \left[f(p(k)) + \frac{f'(p(k))V}{\lambda + r} \right], \tag{14}$$

where $f(p) = F'(p)$ is the density function of the default rate and $f'(p)$ is its derivative. Whilst the first term of (14) is nonnegative (since $f(p(k))$ is a density), the second term can either be positive (if $f'(p(k)) > 0$) or negative (if $f'(p(k)) < 0$). Thus $G(k, V)$ is not, in general, a convex or a concave function of k, which implies that we may have both corner and interior solutions. However, since $F(p(k_{\max})) = F(1) = 1$ and $f(p(k_{\max})) = f(1) = 0$, our assumption $\delta > c$ together with (13) implies that the derivative of $G(k, V)$ with respect to k evaluated at $k_{\max}$ is always negative, so a corner solution with $k = k_{\max}$ can be ruled out. Therefore, the only possible corner solution is $k^* = 0$.

If an interior solution exists, it would be characterized by the first-order condition $\partial G/\partial k = 0$ and the second-order condition $\partial^2 G/\partial k^2 < 0$. Differentiating the first-order condition and taking into account the definition (9) of the franchise value V gives

$$\frac{\partial k^*}{\partial x} = -\left(\frac{\partial^2 G}{\partial k^2} \right)^{-1} \left(\frac{\partial^2 G}{\partial k \partial x} + \frac{\partial^2 G}{\partial k \partial V} \frac{\partial V}{\partial x} \right),$$

where x is any of the six variables that determine economic capital k^*. Now using

$$\frac{\partial^2 G}{\partial k \partial V} = \frac{(1+c)f(p(k))}{(1+\delta)(\lambda + r)} > 0$$

and the second order-condition $\partial^2 G/\partial k^2 < 0$ we conclude that

$$\frac{\partial k^*}{\partial x} > 0 \quad \text{if} \quad \frac{\partial^2 G}{\partial k \partial x} > 0 \quad \text{and} \quad \frac{\partial V}{\partial x} > 0.$$

The problem is that the sign of $\partial^2 G/\partial k \partial x$ is ambiguous for $x = \overline{p}$, λ and ρ, and that for $x = r$ the sign of $\partial^2 G/\partial k \partial x$ is negative whereas the sign of $\partial V/\partial x$ is positive. Therefore, the only two variables for which the sign of the derivative $\partial k^*/\partial x$ can be derived analytically are the deposit rate c and the cost of capital δ, for which it is easy to show that

$$\frac{\partial k^*}{\partial c} < 0 \quad \text{y} \quad \frac{\partial k^*}{\partial \delta} < 0,$$

that is, the higher the bank's deposit and equity funding costs the lower its economic capital.

B   Uninsured deposits' interest rate

The uninsured deposits' interest rate c is obtained by solving the participation constraint (12) that equates the expected value of the depositors' claim at the end of each period, $E\left[\min\left\{a, (1-k)(1+c)\right\}\right]$, to the gross return that they require on their investment, $1 - k$. This Appendix shows that (12) has a unique solution $c(k) \geq 0$ for all k and that $c'(k) < 0$, except for $k \geq \lambda$ in which case $c'(k) = c(k) = 0$.

Using the definition (1) of a, the participation constraint (12) can be rewritten as

$$
\begin{aligned}
U(c,k) &= E\left[\min\left\{a, (1-k)(1+c)\right\}\right] - (1-k) \\
&= (1-k)\left[(1+c)F(p(k))\text{-}1\right] + \int_{p(k)}^{1}\left[1 + (1-p)r - p\lambda\right]\, dF(p) = 0. \quad (15)
\end{aligned}
$$

When $k \geq \lambda$, (15) together with the definition (3) of $p(k)$ imply $U(c,k) = (1-k)c$, so $c = 0$ is the unique solution.

When $k < \lambda$, given that $p(k) < 1$ for all $c \geq 0$, integrating (15) by parts and making use of the definition (3) of $p(k)$ gives

$$U(c,k) = k - \lambda + (\lambda + r)\int_{p(k)}^{1} F(p)\, dp. \quad (16)$$

Using $F(p(k)) < 1$ and the definition (3) of $p(k)$, (16) implies

$$U(c,k) < k - \lambda + (\lambda + r)(1 - p(k)) = (1-k)c,$$

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so $U(0, k) < 0$. Now differentiating (16) with respect to c gives

$$\frac{\partial U}{\partial c} = (1 - k) F(p(k)) > 0, \tag{17}$$

which implies that the equation $U(c, k) = 0$ will have a unique solution $c(k) > 0$ for all $k \in [0, \lambda)$ if

$$\max_c U(c, k) = k - \lambda + (\lambda + r) \int_0^1 F(p)\, dp \geq 0,$$

that is if

$$k \geq \lambda \int_0^1 [1 - F(p)]\, dp - r \int_0^1 F(p)\, dp = \overline{p}\lambda - (1 - \overline{p})r.$$

But from (11) we have $\overline{p}\lambda - (1 - \overline{p})r = -\mu < 0$, as required. Finally, totally differentiating $U(c, k) = 0$ and using (17) and

$$\frac{\partial U}{\partial k} = 1 + (1 + c) F(p(k)) > 0$$

we conclude that $c'(k) < 0$.

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